A Branch-And-Cut-And-Price Algorithm for the Capacitated Location Routing Problem

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Problem Description

- **Input**
  - Set of Potential Facilities $I$. Each facility $i$ has capacity $b_i$ and opening fixed cost $f_i$
  - Set of Customers $J$. Each customer $j$ has demand $d_j$
    - Vertex set $V = I \cup J$, Edge set $E$
  - Fleet of size $K$. Each vehicle has capacity $Q_k$
  - Direct ride cost $c_e$ for every edge $e \in E$

- **Output**
  - $K$ or less simple tours
  - Demand of customers $J$ is satisfied
  - Vehicles do not depass their capacity $Q_k$
  - Facilities do not depass their capacity $b_i$
  - Total Cost (Fixed Cost + Routing Cost) is minimized
Previous Work

■ Heuristics
  ■ Sequential Methods (Perl and Daskin 1985)
  ■ Iterative Methods (Wu et al. 2002)
  ■ Metaheuristics (Tuzun and Burke 1999)

■ Exact Methods
  ■ Branch-and-bound (Laporte et al. 1986)
  ■ Branch-and-cut (Prodhon 2006, Contardo et al. 2009)
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Assumptions and Consequences

- **Assumptions**
  - Homogeneous Fleet: $Q_k = Q$ for all $k$
  - Homogeneous Facilities: $b_i = b$ for all $i$
  - Symmetric Network: $c_{ij} = c_{ji}$ for every pair of arcs $(i,j), (j,i)$

- **Consequences**
  - Minimum number of vehicles needed to serve $S \subseteq J$: $r(S)$
    - $r(S)$ can be replaced by $k(S) = \left\lceil \frac{d(S)}{Q} \right\rceil$
  - Minimum number of facilities needed to serve $S \subseteq J$: $\rho(S)$
    - $\rho(S)$ can be replaced by $\kappa(S) = \left\lceil \frac{d(S)}{b} \right\rceil$
  - The homogeneous structure of facilities had not yet been exploited!!
Formulations

- Three-index formulation (Perl and Daskin 1985).
  - Weak bounds
  - Tight bounds.
  - Valid Inequalities: Cover Inequalities (COV), \( y \)-Capacity Cuts (\( y \)-CC), \( y \)-Effective Facility Capacity Inequalities (\( y \)-EFCI), Location-Routing Comb Inequalities (LR-COMB), \( y \)-Chain Barring Constraints (\( y \)-CBC), etc.
  - Tighter bounds.
  - Solved by column generation. Pricing problem is a SPPRC (\( k \)-cycle, elementary).
  - Valid Inequalities: Strengthened Capacity Cuts (SCC), Clique Inequalities (CLI).
Variables and Notation

- $\Omega = \{\omega : \omega \in \text{routes}\}$.
- for $i \in I$, $\Omega_i = \{\omega \in \Omega : \omega$ starts and ends at facility $i\}$.
- $\Omega^M = \{\omega \in \Omega : \omega$ serves multiple customers (2 or more)\}$.
- $\Omega^S = \{\omega \in \Omega : \omega$ serves a single customer\}$.
- for $\omega \in \Omega$, $\lambda_\omega$ equal to 1 iff route $\omega$ is selected.
- for $i \in I$, $z_i$ equal to 1 iff facility $i$ is selected.
- There exists a relationship between two-index variables and $\lambda_\omega$’s.
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Valid Inequalities

- Derived from the 2-index formulation
  - $y$-Vehicle Capacity Constraints
  - $y$-Chain Barring Constraints
  - $y$-Effective Facility Capacity Inequalities
  - Location-Routing Comb Inequalities
  - Generalized Large Multistar Inequalities
  - All cuts valid for the CVRP

- Derived from the set partitioning formulation
  - Clique Inequalities

- Strengthened Constraints
  - Strengthened Capacity Cuts (SCC) (Baldacci et al. 2009)
  - $y$-SCC (NEW!)
  - $y$-Strengthened FCI ($y$-SFCI, NEW!)
  - $y$-Strengthened EFCI ($y$-SEFCI, NEW!)
## Capacity Cuts

### 2-index formulation

- **CC**
  \[ x(\delta(S)) + 2y(I : S) \geq 2r(S) \] (1)

- **y-CC**
  \[ x(\delta(S)) + 2y(I : S - S') \geq 2r(S) \] (2)

### Set partitioning formulation

- **SCC (Baldacci et al. 2009)**
  \[ \lambda(\{l \in \Omega : J(l) \cap S \neq \emptyset\}) \geq r(S) \] (3)

- **y-SCC (NEW!)**
  \[ \lambda(\{l \in \Omega^M : J(l) \cap S \neq \emptyset\}) + \lambda(\{l \in \Omega^S : j(l) \in S - S'\}) \geq r(S) \] (4)
Pricing Problem

- Pricing problem is a shortest path under resource constraints (SPPRC). It must be run $|I|$ times.
- Strengthened Constraints ($y$-SCC, $y$-SFCl, $y$-SEFCI) added as resources.
- Elementary (ESPPRC, Feillet et al. (2004), Jepsen et al. (2008)) or without $k$-cycles ($k$-SPPRC, Irnich and Villeneuve (2006)).
- Solved by dynamic programming. Usually the bottleneck in B&C&P algorithms
  - Heuristic accelerations
    - Solve for a small number of customers (found by MST, SPP).
    - Bound somehow the number of labels (paths) kept during the algorithm execution.
  - Exact accelerations
    - Graph reduction using reduced costs on edges from the solution of the 2-index LR.
    - Perform efficient and tight domination rules.
    - Discard useless labels.
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Computational Results

| Instance Name | | | BKS | | | Prodhon (2006) | | | Contardo et al. (2009) | | | This paper | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| q05 | 21 | 5 | 424.9 | 403.7 | 0.5 | 4.99 | 404.5 | 0.4 | 4.8 | 422.9 | 9.54 | 0.5 |
| q06 | 22 | 5 | 585.1 | 577.5 | 0.5 | 1.3 | 576.6 | 0.2 | 1.45 | 585.1 | 3.87 | 0.0 |
| q07 | 29 | 5 | 512.1 | 446.3 | 0.5 | 12.85 | 474.1 | 0.8 | 7.42 | 499.0 | 329.0 | 2.56 |
| q08 | 32 | 5 | 571.9 | 518.2 | 1.2 | 9.39 | 524.8 | 1.2 | 8.24 | 545.2 | 2116 | 4.67 |
| q09 | 32 | 5 | 504.3 | 477.8 | 0.4 | 5.25 | 482.9 | 1.0 | 4.24 | 500.5 | 876.2 | 0.75 |
| q10 | 36 | 5 | 460.4 | 433.2 | 3.2 | 5.91 | 436.4 | 2.0 | 5.21 | 455.0 | 437.5 | 1.17 |

**Table:** Root Relaxation
## Computational Results

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</table>

**Table:** Branch-And-Bound
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Conclusions and Further Research

- Combine in the same model the strength of two-index with the set partitioning formulations
- Provide new valid inequalities that strengthen the bounds
- Bottleneck is pricing problem. We easily find columns of negative reduced cost at the beginning. At the end it becomes much more difficult.
References I


References II


