Models and algorithms for the Transit Network Design Problem

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### Public transportation planning process (Ceder y Wilson, 1986)

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Transit Network Design Problem - TNDP

- to find a set of routes $R$ with their corresponding frequencies $F$ for an urban public transportation system.

$S = (R, F)$ is a solution

$R = \{r_1, r_2, \ldots, r_r\} \subseteq \Omega$

$F = \{f_1, f_2, \ldots, f_r\} \subseteq \mathbb{R}^+$

- minimizing the conflicting objectives
  - passengers discomfort and
  - companies operating cost.
Transit network design problem

- **Objectives:**
  - Users: min eg on-board, transfer and waiting times.
  - Operators: min fleet size.

- **Constraints:**
  - Demand satisfaction
  - Required level of service
  - Resource availability

- **Decision Variables:**
  - Routes & Frequencies

- **Data:**
  - Street network
  - Demand of trips
Network & Demand Models

\( G : (N,E,C); \)

\( N = \{i \in [1..n]\} \) bus stop or centroids,

\( E = \{e (i,j), \text{ connections}\} \)

\( C = \{c_e, \text{ on board time}\} \)

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 0 & 80 & 70 & 160 & 50 & 200 & 120 & 60 \\
2 & 80 & 0 & 120 & 90 & 100 & 70 & 250 & 70 \\
3 & 70 & 120 & 0 & 180 & 150 & 120 & 30 & 250 \\
4 & 160 & 90 & 180 & 0 & 80 & 210 & 170 & 230 \\
5 & 50 & 100 & 150 & 80 & 0 & 250 & 40 & 130 \\
6 & 200 & 70 & 120 & 210 & 250 & 0 & 130 & 120 \\
7 & 120 & 250 & 30 & 170 & 40 & 130 & 0 & 70 \\
8 & 60 & 70 & 250 & 230 & 130 & 120 & 120 & 0 \\
\end{array}
\]

Origin-destination Matrix:

\( D = \{d_{ij}, i,j \in [1..n]\}\)

\( d_{ij} : \text{ number of trips per time unit in a given time horizon} \)

\( S = (R,F), \text{ route } k = (1,2,4,7,4,2,1), \)

\( \text{If } f_k=3/60; 3*100/60 = 5 \text{ buses} \)
TNDP- multi objective model


\[
\begin{align*}
\text{Min} \quad z_1(S) &= \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} (tv_{ij} + tw_{ij} + tt_{ij}) \\
\text{Min} \quad z_2(S) &= \sum_{r_k \in R} f_k t_k
\end{align*}
\]

\[D_0(S) \geq D_0^{\min} \in [0,1] \]

\[D_{01}(S) \geq D_{01}^{\min} \in [0,1] \]

\[f_{\min} \leq f_k \leq f_{\max} \quad \forall f_k \in F \]

\[f_k \geq \frac{\max \phi_k(S)}{w_{\max} Q} \quad \forall f_k \in F \]

Passengers overall time.

Fleet size.

Proportion of Tot demand

- without transfer,

- with at least 1 transfer.

Lower & upper bounds for frequencies.

Max Load factor
TNDP - difficulties

- **High combinatorial complexity**: it is a complex variant of the generalized network design problem [Magnanti and Wang, 1984], which is NP-hard [Israeli and Cedër, 1993]

- Requires an **assignment submodel**: requires a behaviour model of the passengers concerning the routes and frequencies of a given solution, to evaluate the solution.

- **Multi-objective nature**: the existence of conflicting objectives adds complexity to the problem, both in the *a priori* estimation of the relative importance of the objectives, and in the calculation of several solutions with different trade-off levels between the objectives (*aposteriori*). Another estimation of the relative importance of the objectives is the *Interactive one*.
The assignment model

- It is a subproblem of the TNDP, a hard problem by itself.

- One of the existing models is the common lines and transfers model of Baaj and Mahmassani (1990).

- It produces a set of flows $\Phi(S)$ (for a given $S$), which specifies how the demand is distributed among a given set of routes, and suggests feasible frequencies.

**Example of the distribution of the demand of trips**

Assigns passengers to routes, There are different alternatives for the same pair of vertices $i,j$
MOCO - Multi Objective Comb. Optimization  

(Ehrgott y Gandibleux, 2004).

Decision space

Objective space

Feasible set $C = \{S_1, S_2, S_4\}$.
Non dominated $Y_N = \{f(s_1), f(s_2), f(s_4)\}$.
$S_4$ dominates $S_3$.
$S_1, S_2, S_4$ are non dominated.
Pareto Fronts (Erhgott, 2005)

- Optimal Pareto Front (exact solution)
- Subset of the optimal Pareto Front
- Set of non dominated solution (not necessary optimal)

GRASP-TNDP
GRASP-TNDP:  (Mauttone and Urquhart, CASPT 2006)

```
procedure GRASP-TNDP(in parameters, out P);
Calc shortest path between all pair vertices of G
P ← ∅;
for i = 1 to NumIterations do
    tmax ← random unif value in given interval;
    Construction (G,...,tmax, ..., R);
    F ← Initial frequencies;
    S ← (R,F);
    λ ← Random vectors of weights
    Local Search (λ,S,P);
    Delete dominated solutions of P;
end for;
return P;
end GRASP_TNDP;
```

Greedy Randomized Adaptive Search Procedures (Feo & Resende, 1995)
GRASP Multi-objective

- Solutions obtained with the construction procedure, set of routes that cover the demand
- Local Search trajectory
- Solutions obtained with the local search procedure
- Pareto Front
GRASP-TNDP: Construction

PIA- Pair Insertion Algorithm

(Mauttone y Urquhart, 2009).

- Produces a set of routes which satisfies the demand
  
  1. using the shortest path between pairs of vertices with high demand, measured in on-board time;
  
  2. inserting pairs of vertices in already constructed routes in the solution under consideration.
TNDP: PIA algorithm

procedure PIA(in parameters, out R);
  R := 0 ;; initialize variables;
  l :- List of pairs of vertices (i; j) of G with \( \text{dij} \neq 0 \);
  while demand do
    \((u; v)\) :- Select \((i; j)\) with maximum \( \text{dij} \) in \( l \);
    \(r\) :- Create a route with the shortest path between \( u \) and \( v \) in \( G \);
    \(r'\):- Create a route by inserting \( u \) and \( v \) in the most suitable positions in the most
    convenient route \( r'' \) in \( R \), by calling Candidate\((u; v; R; r^0)\);
    if cost\((r)\) < cost\((r') - cost(r'')\) then
      \(R := R \cup \{r\}\);
      Delete from \( l \) pairs of vertices whose demand is covered directly by \( r \);
    else
      \(R := R \cup \{r^0\} \setminus \{r''\}\);
      Delete from \( l \) pairs of vertices whose demand is covered directly by \( r' \);
    end if;
    Update demand;
  end while;
  Filter routes in \( R \);
  return \( R \);
end PIA;
Pair Insertion Algorithm: example

Empty set of routes
Pair Insertion Algorithm: example

Selected pair of vertices with high demand
Pair Insertion Algorithm: example

Construction of a new route with the shortest path measured in terms of on board time:
Pair Insertion Algorithm: example

Another pair with high demand is selected.
Pair Insertion Algorithm: example

The selected pair of vertices is inserted in the existing route.
Another pair of vertices with high demand is selected.
Pair Insertion Algorithm: example

A convenient new route is created
Case of study: Rivera city

Rivera instance:  http/www.fing.edu.uy/~mauttone/tndp

- 65000 inhabitants - Medium –small instance
- 13 bus-lines.
- Mean length of routes: 13,6 Km.
- Frequencies 20, 30, 40 y 60 minutes.
- **Fleet size (buses): 23.**
- Demand: 13360 trips per day (Monday-Friday).
- On-board Bus Survey done in 2004. (based on Stopher *et al.* (1986)).
- Household Survey done in 2007. (Celina Gutierrez)
Results

![Graph](image)

- Operators' cost, $Z_2(S)$
- Users' cost, $Z_1(S)$

Legend:
- **GRASP TNDP**
- **Rivera**
A decision support system that helps researcher to:
- create and maintain different projects;
- experiment with new algorithms and models;
- evaluate solutions;
- support data processing by handling geographical and origin-destination data and O-D matrixes.
- visualize both numerical and geographical information (medium small instances);

- It is developed within our project using the open source mapwindow. ([http://www.mapwindow.org/](http://www.mapwindow.org/)).
- It is compatible with the ESRI format.
IgoR-tp: assembling the demand
IgoR tp – Algorithms

Nueva Configuración de Algoritmo

Nombre: Heuristica_1
Ubicación del Algoritmo: F:\caso Rivera
Cantidad de pasajeros: 10, 15, 20

Nueva Configuración de Algoritmo

Parámetros

Nombre de la solución: Sol_heuristica_2
Selezione una configuración de flota

Generar Recorridos...

Complete los parámetros necesarios

Demanda directa: 0.8
Tamaño de flota: 23
Bus Lines System of Rivera
Alternative Solution  (Lucas Faccelo, 2007)
Ongoing and future work

- Up to now, IgoR-TP is for research purposes, however, it may be extended with features oriented to the transit planner.

- We are working with new models and algorithms for the TNDP.
  - Bi-level mathematical programming formulation and exact calculation of very small cases.
  - Our focus is more on the definition of routes.

- Colaborate with the Municipalities as far as possible…
References

Thank you

Engineering Faculty

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