Routing Problems with Profits

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Some facts

- In Europe a large percentage of trucks travel with no load (30%-40%)
- Average load around 1/3 of the capacity
- Small size of the carriers
- Geographic dispersion of the customers of a carrier
- Low level of cooperation among carriers
- Customers service level
Opportunities

Remark: Competition pushes carriers (and shippers) towards efficiency

Possibilities/opportunities

- Increase of the size of the carriers
- Cooperation among carriers
- Electronic services that increase the efficiency
Opportunities

Remark: Competition pushes carriers (and shippers) towards efficiency

Possibilities/opportunities

- Increase of the size of the carriers
- Cooperation among carriers
- Electronic services that increase the efficiency
OR literature on electronic transportation services

- Mathematical programming models to determine the winner of auctions for long-term contracts
- Few attempts to provide carriers with decision support tools in electronic auctions
Decision support models

- Given a set of contracts, how to select the most profitable spot loads to bid on?
- Given a set of contracts, how to select potential lanes to bid on?
Decision support models

- Given a set of contracts, how to select the most profitable spot loads to bid on?
- Given a set of contracts, how to select potential lanes to bid on?
Crucial decision: To bid or not to bid?

The decision depends on the fleet, on costs and profits
Crucial decision: To bid or not to bid?

Routing problems with profits
TSP with profits

- Orienteering problem (OP)
  - Max collected profit (constraint on time)
- Prize-collecting TSP
  - Min traveling cost (constraint on profit)
- Profitable tour problem (PTP)
  - Max (collected profit – traveling cost)

One uncapacitated vehicle - Feillet, Dejax, Gendreau (2005)
## VRP with profits

### Team Orienteering Problem (TOP)
- Max collected profit
- Constraint on time for each vehicle – uncapacitated vehicles

### Capacitated TOP (CTOP)
- Max collected profit
- Constraint on time and capacity for each vehicle

### Capacitated PTP (CPTP)
- Max (collected profit - traveling cost)
- Constraint on capacity for each vehicle
The Team Orienteering Problem (TOP)

Potential customers – associated profit
Fleet of vehicles - maximum time available for each tour

Objective: Maximize the total profit
The TOP

- A fleet of $m$ vehicles
- A set of potential customers
- A symmetric time distance $c$ associated to each edge $(i,j) \in E$
- A profit $p$ associated to each potential customer
- A time limit $T_{max}$ on the route duration of each vehicle

Note: The profit of each potential customer can be collected by one vehicle at most

Objective: Maximize the total profit

TOP is also called Multiple Tour Maximum Collection Problem
The literature

Definition of the TOP

- Butt, Cavalier (1994)
- Chao, Golden, Wasil (1996)

Heuristic Algorithms

- Tsiligirides (1984): Heuristic algorithm for the OP
- Chao, Golden, Wasil (1996): Heuristic algorithm for the TOP
- Tang, Miller-Hooks (2005): Tabu search + Adaptive memory
- Archetti, Hertz, Speranza: (2007) Tabu search and VNS
- Ke, Archetti, Feng (2008): Ant colony

Exact Algorithms


**Corollary:** TOP is NP-hard
Capacitated TOP (CTOP)

- Maximize total collected profit
- Time and capacity constraint on each vehicle

2 vehicles
Q=5
Tmax=5

all customers have demand 2
Capacitated TOP

Capacitated TOP (CTOP)
- Maximize total collected profit
- Time and capacity constraint on each vehicle

2 vehicles
Q=5
Tmax=5
all customers have demand 2

Optimal o.f.=12 (profit)
Algorithms for CTOP

- Exact approach: Branch-and-price
- Heuristics: Tabu search and variable neighborhood search
- Tests on modified benchmark instances
Branch-and-price for the CTOP

Boussier, Feillet, Gendreau (2007)

Column generation to solve the Master Problem

\[
\begin{align*}
\max & \quad \sum_{r_k} c_k x_k \\
\sum_{r_k} a_{ik} x_k & \leq 1 \quad i \in V - \{1, n\} \\
\sum_{r_k} x_k & \leq m \\
x_k & \in \{0,1\} \quad \forall r_k
\end{align*}
\]

\(\lambda_i\) and \(\lambda_1\)

Branching phase

Master Problem = linear relaxation
Column generation - CTOP

Restricted Master Problem on a subset of routes

Subproblem to identify good routes

Elementary shortest path problem with resource constraints
- Graph G with cost $\lambda_i - p_i$ on arc (i,j)
- Two resources: capacity and time
Branching phase - CTOP

1. Branching on customers (when a customer is visited a fractional number of times)
   Two branches:
   - visit customer i
   - do not visit customer i

2. Branching on arcs (when an arc is traversed by a fractional number of vehicles)
   Three branches:
   - do not visit customer i
   - visit i followed by j
   - visit i not followed by j
Tabu search

- Choose an initial solution $s'$; set $TL = \emptyset$ (tabu list); set $s^* = s'$ (best solution)

- Repeat until a stopping criterion is met
  - Determine a best solution $s''$ in $N(s')$ such that either $s''$ is not in $TL$ or $s''$ is better than $s^*$
  - If $s''$ is better than $s^*$ then set $s^* = s''$
  - Set $s = s''$ and *shake* $s$ to obtain $s'$ and update $TL$

*jump*  
*internal tabu search*
Variable Neighborhood Search

- Choose an initial solution \( s \); set \( k = 1 \)
- Repeat until a stopping criterion is met
  - **shaking:** Generate \( s' \) at random in \( N^{(k)}(s) \)
  - **local search:** Apply a local search on \( s' \). Let \( s'' \) be the resulting solution
  - **update:** If \( s'' \) is better than \( s \) then set \( s = s'' \) and \( k = 1 \), else set \( k = (k \mod \ k_{\text{max}}) + 1 \)
Metaheuristics for the CTOP

Two generalized tabu search algorithms and one VNS algorithm

Main concepts

For a solution $s$
- $\text{RCTOP}(s)$ is the set of $m$ most profitable routes in $s$
- $\text{RNCTOP}(s)$ is the set of remaining routes in $s$

A **solution $s$ is feasible** if each route is feasible
A **solution $s$ is admissible** if each route in $\text{RNCTOP}(s)$ is feasible

If an admissible solution has non-feasible routes, these routes belong to the set of the most profitable routes.
An admissible solution

Example with 3 vehicles

Keeps the non-profitable customers structured in routes.
Jumps

First kind of jump:
k customers at random are moved from RNCTOP to RCTOP

Second kind of jump:
k customers at random are moved from RCTOP to RNCTOP and are replaced by a set of more profitable customers that are moved from RNCTOP to RCTOP
First kind of jump
Second kind of jump

The solution is made admissible, if necessary
The internal tabu search
Two kinds of moves

1-move: customer c is moved from its route to another route r. Route r can be an empty route.

swap-move: Let c and c' be two customers on two different routes. Customers c and c' are swapped.
The internal tabu search

- Maximum number of iterations
  - Long: about one minute (LONG_TABU)
  - Short: about one second (SHORT_TABU)

- Strategy
  - Feasible: The search space contains all feasible solutions
  - Penalty: The search space contains all admissible solutions
Heuristics

- (GENERALIZED) TABU FEASIBLE
  - Few jumps and LONG_TABU with feasible strategy
- (GENERALIZED) TABU ADMISSIBLE
  - Few jumps and LONG_TABU with penalty strategy
- VNS (FEASIBLE)
  - Many jumps and SHORT_TABU
10 Christofides, Mingozi, Toth (1979) instances taken from the VRP library with both capacity and time constraints.

Number of vertices: from 51 to 200.

Starting from these 10 basic instances we constructed three sets of instances:

**Set I - Original instances**
large number of vehicles

**Set II - 90 instances**
a smaller number of vehicles (m=2,3,4) and various values of Q and Tmax

**Set III - 30 instances**
obtained by only changing the number of vehicles (m=2,3,4) with respect to the original values
(very hard to solve – heuristic solutions only)
<table>
<thead>
<tr>
<th>Instance</th>
<th>B&amp;P</th>
<th>VNS</th>
<th>Tabu Feasible</th>
<th>Tabu Admissible</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p*</td>
<td>CPU</td>
<td>p</td>
<td>CPU</td>
</tr>
<tr>
<td>3</td>
<td>1409</td>
<td>41</td>
<td>1409</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>761</td>
<td>2</td>
<td>761</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
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<td>2</td>
<td>1327</td>
<td>0</td>
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<tr>
<td>9</td>
<td>1164</td>
<td>2064</td>
<td>5006</td>
<td>0,00</td>
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<tr>
<td>10</td>
<td>1735</td>
<td>3048</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
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<td>1710</td>
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</tr>
<tr>
<td>16</td>
<td>588</td>
<td>2968</td>
<td>12310</td>
<td>0,00</td>
</tr>
</tbody>
</table>

Time limit of the B&P = one hour

Easy instances
CTOP – Set II

90 instances

60 instances solved to optimality within the time limit of one hour

<table>
<thead>
<tr>
<th></th>
<th>VNS</th>
<th>Tabu feasible</th>
<th>Tabu admissible</th>
</tr>
</thead>
<tbody>
<tr>
<td>average</td>
<td>time (in sec.)</td>
<td>average error</td>
<td>time (in sec.)</td>
</tr>
<tr>
<td>error</td>
<td>0.07</td>
<td>0.06</td>
<td>0.21</td>
</tr>
</tbody>
</table>

VNS gives 79 best solutions, 53 proved optima
On 15 instances the VNS finds a solution better than the best provided by the B&P within the time limit
### CTOP – Set III

<table>
<thead>
<tr>
<th></th>
<th>VNS</th>
<th>Tabu feasible</th>
<th>Tabu admissible</th>
</tr>
</thead>
<tbody>
<tr>
<td>average error</td>
<td>0,00</td>
<td>0,29</td>
<td>0,22</td>
</tr>
<tr>
<td>time (in sec.)</td>
<td>1319</td>
<td>388</td>
<td>260</td>
</tr>
</tbody>
</table>

30 instances

Errors with respect to the best solution found

VNS gives all best solutions
Capacitated PTP (CPTP)
- Maximize (total collected profit - total traveling cost)
- Capacity constraint on each vehicle

2 vehicles
Q=5

all customers have demand 2

traveling cost and time

profit
Capacitated PTP

Capacitated PTP (CPTP)
- Maximize (total collected profit - total traveling cost)
- Capacity constraint on each vehicle

2 vehicles
Q=5

all customers have demand 2

Optimal o.f. = 18(profit) - 12(cost) = 6
Algorithms for the CPTP

**Exact approach:** Branch-and-price for CTOP adapted to CPTP

**Heuristics:** Two Tabu Search and one Variable Neighborhood Search for CTOP adapted to CPTP
- Different measure of infeasibility: \( \max\{D(r)-Q, 0\}^2 \)
- Pay attention to routes with negative values
### CPTP – Set I

<table>
<thead>
<tr>
<th>Instance</th>
<th>VNS</th>
<th>Tabu Feasible</th>
<th>Tabu Admissible</th>
</tr>
</thead>
<tbody>
<tr>
<td>instance</td>
<td>v</td>
<td>CPU</td>
<td>%</td>
</tr>
<tr>
<td>3</td>
<td>663,98</td>
<td>400</td>
<td>0,00</td>
</tr>
<tr>
<td>6</td>
<td>258,97</td>
<td>22</td>
<td>0,00</td>
</tr>
<tr>
<td>7</td>
<td>534,81</td>
<td>198</td>
<td>0,00</td>
</tr>
<tr>
<td>8</td>
<td>663,98</td>
<td>400</td>
<td>0,00</td>
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<tr>
<td>9</td>
<td>1189,33</td>
<td>3810</td>
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<tr>
<td>10</td>
<td>1773,65</td>
<td>1384</td>
<td>0,00</td>
</tr>
<tr>
<td>13</td>
<td>284,71</td>
<td>219</td>
<td>0,00</td>
</tr>
<tr>
<td>14</td>
<td>890,44</td>
<td>145</td>
<td>0,00</td>
</tr>
<tr>
<td>15</td>
<td>1168,63</td>
<td>3298</td>
<td>0,00</td>
</tr>
<tr>
<td>16</td>
<td>1791,78</td>
<td>13047</td>
<td>0,00</td>
</tr>
<tr>
<td>average</td>
<td>0,03</td>
<td>1,32</td>
<td>1,81</td>
</tr>
</tbody>
</table>

No optimal solution found within one hour
CPTP – Set II

90 instances

53 instances solved to optimality within the time limit of one hour

<table>
<thead>
<tr>
<th>VNS</th>
<th>Tabu feasible</th>
<th>Tabu admissible</th>
</tr>
</thead>
<tbody>
<tr>
<td>average error</td>
<td>time (in sec.)</td>
<td>average error</td>
</tr>
<tr>
<td>0,00</td>
<td>484</td>
<td>0,39</td>
</tr>
</tbody>
</table>

VNS gives all best solutions, 45 proved optima
On 23 instances the VNS finds a solution better than the best provided by the B&P within the time limit
CPTP – Set III

30 instances

<table>
<thead>
<tr>
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<th>Tabu admissible</th>
</tr>
</thead>
<tbody>
<tr>
<td>average error</td>
<td>time (in sec.)</td>
<td>average error</td>
<td>time (in sec.)</td>
</tr>
<tr>
<td>0.21</td>
<td>786</td>
<td>1.35</td>
<td>143</td>
</tr>
</tbody>
</table>

Errors with respect to the best solution found

VNS gives 25 best solutions
More profit with same fleet?

- We know that split deliveries may reduce costs and number of routes in VRP.
- Can we take advantage of split deliveries to increase the profit in CTOP and CPTP? How much?
Capacitated TOP with Split Deliveries

SDCTOP

- Maximize total collected profit
- Time and capacity constraint on each vehicle

A customer may be visited by multiple vehicles
Capacitated PTP with Split Deliveries

SDCPTP
- Maximize (collected profit – traveling cost)
- Capacity constraint on each vehicle

A customer may be visited by multiple vehicles
An example

2 vehicles
Q=5

Profit of each customer = 10

CTOP

Profit = 20

SDCTOP

Profit = 30
The k-split cycles

**Definition:** Given any subset of k customers 1, 2, ..., k and k routes. Route 1 visits customers 1 and 2, route 2 visits customers 2 and 3, ..., route k−1 visits customers k−1 and k, and route k visits customers k and 1. The subset of customers 1, 2, ..., k is called a k-split cycle.

Some properties

*Properties*: If the cost matrix satisfies the triangle inequality, then there exists an optimal solution to the SDCTOP and SDCPTP where:

- there is no k-split cycle (for any k);
- no two routes have more than one customer with a split delivery in common;
- the number of splits is less than the number of routes.

*These properties are useful in the design of solution algorithms*
Max. gain with split deliveries

\[
\frac{CTOP}{SDCTOP} \geq \frac{1}{2} \quad \text{and the bound is tight}
\]

\[
\frac{CPTP}{SDCPTP} \geq \frac{1}{2} \quad \text{and the bound is tight}
\]

Archetti, Bianchessi, Hertz, Speranza, *work in progress*
Tightness of the bound

$m$ customers on each circle
$m$ vehicles

$Q \geq 2m$

optimal solution CTOP:
direct trips

$CTOP = mp$

optimal solution SDCTOP:
2 customers along the radius with $m-1$ vehicles and 1 additional tour ($m-1$ customers on the inner circle are served twice)

$SDCTOP = 2(m-1)p$

Profit of each customer is $p$
Decision support models

- Given a set of contracts, how to select possible spot loads to bid on?
- Given a set of contracts, how to select potential lanes to bid on?
Crucial decision: To bid or not to bid?

Arc routing problems with profit
Arc routing problems with profits

Prize Collecting Rural Postman Problem
Max collected profit - cost (one vehicle, no constraint)

Profitable Arc Routing Problem
Max collected profit - cost (constraint on time on each vehicle)
Feillet, Dejax, Gendreau (2005)
The Capacitated Arc Routing Problem with Profits

- Undirected graph $G(V,E)$, travel time associated to each edge
- Subset of profitable edges with demand and profit
- A fleet of $m$ vehicles with capacity and time limit

**OBJECTIVE**
Maximize the total collected profit

Conclusions

- Routing problems with profits is an interesting class to explore.
- Branch-and-price solves instances of reasonable size and is useful to compute errors of heuristics.
- Variable Neighborhood Search with internal Tabu Search offers an excellent trade-off between solution quality and computational time.