Heuristic Solution Procedures for Covering Path Problems

Liyan Hua, John Current, and David Schilling
Fisher College of Business,
The Ohio State University
Columbus, OH, 43210, USA
Outline

• Motivation and Literature
• Model
• Decomposition method
• Heuristic comparison
• Conclusion
Motivation

• Close relationship between facility location and transportation network design
• Water pipeline:
  - Different cost of major pipeline and sub pipelines
  - Shortest path of major pipe lines
  - Maximum coverage by sub pipelines
  - Water station capacity limit
• Other application:
  - Subway line, rail line design
  - Gas, water pipeline design
  - Power transmission design
  - Warehouse and distribution network
can add more for motivation
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Underlying Problem

K(D,CAP): location(demand, capacity)

Facility location
Source and destination node
Network arc
Demand allocation
Path arc
Related literature: Maximum covering-shortest path problem (MCSP)

• Current, ReVelle and Cohon (1985) first studied the un-capacitated version of the model

• Method: LP-relaxation with branch and bound
Related literature: Path problems

- **Shortest covering path problem** (SCPP, Current, ReVelle and Cohon, 1984).

- **Minimum covering /shortest path problem** (MinCSP, Current, ReVelle and Cohon, 1988; Coutinho-Rodrigues, Climaco, and Current 1999)

- **Median shortest path problem** (MSPP, Current, ReVelle and Cohon, 1985)

- **Bounded length median path problem** (BLMPP, Lari, Ricca and Scozzari, 2008)
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Related literature: Tour problems

• **Covering salesman problem** (CSP, Current and Schilling, 1989)

• **Median tour problem** (MTP) and **Maximal covering tour problem** (MCTP) (Current and Schilling, 1994)

• **Bi-objective covering tour problem** (Gendreau, Laporte and Semet, 1997)
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Related literature: Tree problems

• Minimum cost covering tree problem (MCCS, Hutson and ReVelle, 1989)
• Maximum utilization subtree problem (MUSP, George, ReVelle and Current, 2002)
• Minimum cost partial covering subtree of a tree (Kim, Lowe, Ward and Francis, 1990)
• Maximal covering tree problem on a tree network (Church and Current, 1993)
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Our Contributions:

• Facility capacity constraints are introduced to the maximum covering/shortest path problem
• Decomposition method for the general maximum covering/shortest path problem (MCSP)
• Heuristic methods for the capacitated maximum covering sub-problem
• Heuristic comparisons
Model: Variables

• $x_{ij} = 1$ if arc $i-j$ is in the path
  0 otherwise

• $y_{jk} = 1$ if demand at location $k$ is assigned to facility at $j$
  0 otherwise

• $Z_j = 1$ if there is a facility in location $j$
  0 otherwise
Parameters

• n: total number of nodes in the network
• p: number of facilities to be built
• $d_{ij}$: the distance from node i to j
• $a_{jk} = 1$ if demand at node k is within coverage distance of facility at location j (=0 otherwise)
• $h_k$: the demand at node k
• $M_i = \{j \mid (i,j) \text{ is an existing arc}\}$
• $N_j = \{i \mid (i,j) \text{ is an existing arc}\}$
• $c_j$: the capacity of a facility at location j
Overall model-CMCSP
Decomposition method

• **Path problem:**
  - kSP algorithm to generate paths with nondecreasing order of length

• **Capacitated maximum covering problem (CMCP):**
  - lagrangian relaxation
  - Tabu search
Shortest path sub-problem
k-Shortest Path Problem (kSP) – Solution methodology

- **Reference:**
  Azevedo, Costa, Madeira and Martins (1993)
  Azevedo, Madeira, Costa, Martins and Pires (1994)

- **Method:** path deletion algorithm
  Remove the path and keep all other possible paths by creating extended network

- **Speed:** for Randomly generated network with $10^4$ nodes and $10^5$ arcs, the next 100 shortest paths besides the shortest path can be determined in $10^{-1}$ seconds
Capacitated maximum covering problem
lagrangian relaxation
Capacitated maximum covering problem – lagrangian relaxation
Capacitated maximum covering problem – lagrangian relaxation: upper bound

Break to knapsack problems

• Ignore constraint 31 for a moment
• Break the relaxed problem to separate sub-problems
• Pick the $p$ sub-problems which yield the greatest $p$ objective values, and assign facilities to these sites (set $z_j=1$)
Capacitated maximum covering problem – lagrangian relaxation: upper bound

- Sub-problem at facility location $j$ reduces to a knapsack problem

- Solve by greedy add heuristic ranked by the ratio of \((h_k - \lambda_k)/h_k\)

  If \((h_k - \lambda_k)/h_k \leq 0\), then set \(y_{jk} = 0\)

  \(K_{jy}\) is the set of demand nodes within coverage distance of facility at \(j\) \((a_{jk} = 1)\), and which gives positive \((h_k - \lambda_k)/h_k\)
Capacitated maximum covering problem – lagrangian relaxation : lower bound heuristic

- Eliminate coverage by factional demand assignment \(0 < y_{jk} < 1\)
- Remove multiple coverage: keep the closest facility, or keep the facility with min remaining capacity
- Assign uncovered demand (in nonincreasing order of demand) to the facilities with remaining capacity (in nonincreasing order of remaining capacity)
- For the facility site with remaining capacity (in nonincreasing order of remaining capacity), explore the possibility of exchanging a satisfied demand (nondecreasing order of demand) with an unsatisfied demand (nonincreasing order of demand)
- Repeat the previous two steps one more time each
lh14 may delete some details
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Capacitated maximum covering problem – lagrangian relaxation

• **Multiplier update**: subgradient optimization
• **Stopping rule**:
  1. The number of iterations reaches a limit (500)
  2. Upper bound is within 0.01% of the best lower bound
  3. Constant multiplier $\alpha < 0.0001$ (constant in step size calculation)
  4. All sub-gradients are 0
might change some of the standards.
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this slide is not needed, just keep for reference
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Capacitated maximum covering problem – Tabu search

- To our knowledge, there is no existing research on meta-heuristic for the capacitated maximum covering problem (CMCLP)

- 3 kinds of initial solutions:
  - completely random generated solution
  - greedy generated solution
  - greedy random adapted solution
randomly pick each additional facility. For each of the selected facility, randomly pick the demand. If capacity violated, drop the current demand and check the next randomly selected demand, till all demand has been considered.

same as completely random generated solution. But the order of facilities and demand to be added are different. Order the facility site in nonincreasing order of the minimum of capacity and total uncovered demand by each facility. Break tie by weighted total demand. Add facilities in this order up to p. When a facility is added, select the demand within coverage distance in nonincreasing order of \( \frac{h_k}{nF_k} \) (demand divided by the number of facilities covering this demand).

This is a combination of the previous two methods.
We have a reserve value for the lower bound for both facility selection and demand selection. Random selection is from the facility (demand) with ranking value greater than this observe value.

Reserve value = \( \min + \alpha (\max - \min) \).
Neighborhood structure (in different levels):

A. one-interchange: exchange one selected facility with one unselected facility

Exchange selected facility 2 & unselected facility 3
Capacitated maximum covering problem – Tabu search

Neighborhood structure (in different levels):

B. Client shift: shift one satisfied demand from one to another open facility

Shift demand 3 from selected facility 2 to 1
Capacitated maximum covering problem – Tabu search

**Neighborhood structure** (in different levels):

C. Demand Reallocation: release the assigned demand of a facility, and reassign demand to it

Release assigned demand for facility 1 and reassign demand to it
Capacitated maximum covering problem – Tabu search

**Neighborhood structure** (in different levels):

**D. Demand Addition:** add one or more demand to the open facilities

Add unsatisfied demand 4 and 5 to open facility 1
Capacitated maximum covering problem – Tabu search

Get an initial solution (one of the 3 methods)

Begin outer loop
   A. one-interchange loop
   B. client-shift loop
   C. demand reallocation loop
   D. demand addition step

End outer loop

Improve the solution before next outer loop
this embedded inner loop within outer loop Tabu search is new

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Capacitated maximum covering problem – Tabu search

Stopping rule:

• Do Tabu search for A until no improvement or number of iterations reaches an upper limit
• Do Tabu search for B with the same stopping rule
• Do Tabu search for C with the same stopping rule
• Do D for one time, then go to the next outer loop
• Start over from A to D, until no improvement for the best solution from the last outer loop within a number of loops or the total number of loops reaches an upper limit
Capacitated maximum covering problem – Tabu search

- Demand assignment sub-problem in neighborhood search:
  - Heuristic demand allocation
  - Relaxation with penalty cost for violated capacity

- 6 combination for Tabu search
  - heuristic/penalty demand assignment
  - 3 initial solution generation methods
Capacitated maximum covering problem – Tabu search

One-interchange heuristic method:

- Remove a selected facility at s
  - release the demand from s
  - if any of these demand can be covered by other open facilities, assign the demand to one of them (non-increasing order of remaining capacity)

- Add an unselected facility at t
  - Add unsatisfied demand within coverage distance of t

- Select the best one interchange among all possible interchanges
Capacitated maximum covering problem – Tabu search

- Capacitated maximum covering problem – Tabu search

**RCMCLP:**

\[
\begin{align*}
\text{Maximize} & \quad \Pi_2 = \sum_{j \in P} \sum_{k=2}^{n-1} h_{kj}y_{jk} - \text{Penalty}(Y, Z) \\
\text{St} & \quad \sum_{j \in P} z_j = p \\
& \quad \sum_{j \in P} y_{jk} \leq 1 \quad \forall k = 2, ..., n - 1 \\
& \quad y_{jk} \leq a_{jk}z_j \quad \forall j \in P, k = 2, ..., n - 1. \\
& \quad \sum_{j \in P} \sum_{k=2}^{n-1} h_{kj}y_{jk} \leq \sum_{j} c_jz_j \\
& \quad y_{jk} = (0, 1) \quad \forall (j, k) \\
& \quad z_j = (0, 1) \quad \forall j = 1, 2, ..., n.
\end{align*}
\]

\[
\text{Penalty}(Y, Z) = \sum_{j \in J} \rho_jv_j
\]

\[
v_j = \max\{\sum_{k=2}^{n-1} h_{kj}y_{jk} - c_j, 0\}
\]
Capacitated maximum covering problem – Tabu search

- Penalty coefficient initialized to be $\rho_j = 2$
- Multiplied by $2 \left[ \frac{n_{\text{in}}}{5} - 1 \right]$ in every 10 iterations
  - double if all the previous 10 iterations gives infeasible capacity constraint for facility $j$
  We assign penalty factor for each of the facility sites
if all previous 10 iterations gives feasible solution, rho will halve.
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this is an improvement from the previous paper
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Capacitated maximum covering problem – Tabu search

Construct a feasible solution for penalty method in the end

- For facilities with positive capacity violation, drop the demand in nonincreasing order until violation reaches 0
- do one-interchange, client shift, demand interchange and demand addition similar to the Tabu search. The only difference is to move to the best neighborhood if it improves the solution.
we have outer loop and inner loop. Demand addition is used after the 3 inner loops before a new outer loop starts.

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Capacitated maximum covering problem – Tabu search

- 3 Tabu list for one-interchange, client-shift, demand interchange
- Move decision: move to the best neighborhood in each of the 3 neighborhoods if nonTabu
- Aspiration: if the new solution is better than the best solution in history, make the move even if it is on the Tabu list
- Intensification (only in one-interchange): bring in the facility which has the highest frequency to be selected in one-interchange (in every 20 iterations)
- Diversification (only in one-interchange): bring in the facility which has been out for the longest time (in every 10 iterations)
Overall algorithm stopping rule – kSP /capacitated maximum covering

• No further path can be found (length of the shortest path goes to infinity).

• 92% of total demand has been covered.
paths exhausted
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can change 90% to 95% or more.
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Computation Complexity

- K-shortest path $O(km)$
- Capacitated maximum covering $O(n^2 \log n)$
- Overall: $O(n^2 \log n)$
Output for Lagrangian relaxation
(SP+first 7 paths)

<table>
<thead>
<tr>
<th>k</th>
<th>CMC obj</th>
<th>CMC Ubound</th>
<th>CMC Gap</th>
<th>kSP Time (secs)</th>
</tr>
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<tbody>
<tr>
<td>985</td>
<td>4,510,600</td>
<td>7,110,606</td>
<td>57.6421</td>
<td>0.565</td>
</tr>
<tr>
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</tbody>
</table>
Output for Tabu search

- Random generated initial solution (SP+first 7 paths)
Coverage – distance tradeoff for TS

Red dots are undominated points
can not get the red line by software. Get the scatterplot with dots by excel, then draw the undominated line by hand.
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Comparison of Tabu search and Lagrangian relaxation

• Both algorithms stopped when $k = 47$ paths as 92% of the demand covered
• TS gives better objective values in some cases
Coverage/distance plot for Tabu search and lagrangian relaxation
Coverage/distance plot for Tabu search and lagrangian relaxation
Percentage gap from upper bound

TS CMC gap
LR CMCGap
upper bound is not very good due to multiple coverage
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Comparison of Tabu search and Lagrangian relaxation - more

Impact of facility capacity:

• Small capacity/or facilities with different capacity, Tabu search is better
• For medium/large capacity, approximately the same, lagrangian relaxation could be better
More observations

• Larger coverage distance / capacity
  → stops with fewer paths
• Full network stops faster than partial network
• Large capacity → smaller % gap of solution from upper bound
Summary contributions

• Add capacity to maximum covering/shortest path problem
• Decomposition method
• Heuristics for capacitated maximum covering problem
• Heuristic comparisons
Future research

• Hybrid heuristics for capacitated maximum covering sub-problem
• Different decomposition: first solve the covering problem, then solve a constrained path problem
• Extend to other network structures (e.g., tours, trees)
• Extend to other location objectives (median, minimax)
Muchas gracias!

Questions?

Please call

Liyan Hua

1-614-292-3166