Routing and scheduling in liner shipping

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Outline of presentation

- Liner shipping
  - What is it?
  - Why is it important?
- Liner shipping and the literature
- Weekly frequency
- The heuristic
- Future work
Liner shipping – what is it?

Characteristics of liner shipping:
- Maritime transportation of containerized cargo
- Published time schedules
- Demand dependent on service provided
- Closed routes
- Ships rarely empty
- Transshipments

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<th>Terminal Name</th>
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Liner shipping – why is it important?

- Heavy pressure on road and air networks
- 9.6% growth per year from 1990 to 2005 in container tonnage
- Highly fragmented industry; mergers and acquisitions expected
- Little research to support the area
Literature
- modeling liner shipping

- All feasible schedules generated
  - Fagerholt and Lindstad; Optimal policies for maintaining a supply service in the Norwegian sea. Omega, 2000.

- Limited voyage length
Literature
- modeling liner shipping

- Upper limit on number of ships
  - Agarwal and Ergun; Ship scheduling and network design for cargo routing in liner shipping. Transportation Science, 2008.

- No service requirements
Requirement for research

Routing and scheduling of a liner fleet and cargo routing

While ensuring that the model and solution method:
- can handle a global network size
- has speed as a decision variable
- has no time horizon
- has a service requirement (frequency)
Ships and routes in Liner Shipping

- Several ship types
- One route - one type
- One route per ship
Weekly frequency and how it is obtained

- The duration of a route in weeks is equal to the number of ships required.
- a) 4 week roundtrip with 4 ships at a speed of 19.4 knots
- c) 5 week roundtrip with 5 ships at a speed of 14.0 knots
Weekly frequency and the effect on the model

- Routes
  - Initially 1 ship per route
  - Fix speed and time required
  - Time required = required # ships

- Costs
  - Routes with varying duration
  - Costs on weekly basis
  - Time horizon irrelevant
Formulation and solution method

- Flow model
  - NP hard
  - Non-linear objective function and constraints

- Heuristic solution method
  - Generalized set partitioning based heuristic
  - Optimize ‘master’ problem via CPLEX
Heuristic – schematic presentation

1. Generate support columns.
2. Generate initial columns.
3. Solve LP-master
   - Is there any surplus? Yes: Generate columns. No: Is the solution integer?
     - Yes: Final solution found.
     - No: Fix variable closest to 1 to 1.
Master problem  
- formulation

\[
\begin{align*}
\text{min} & \quad \sum_{r \in R} C_r y_r \\
\text{s.t.} & \quad \sum_{r \in R} L_{mrq} i y_r = 1, \quad \forall m \in M, q = 0, i = 0, \\
& \quad \sum_{r \in R} L_{mrq} i y_r = 0, \quad \forall m \in M, q \in Q_m \setminus \{0\}, i \in I_{mq} \setminus \{0\}, \\
& \quad \sum_{r \in R} UV_r y_r \leq AV_t, \quad \forall t \in T, \\
& \quad L_{mrq} \in \{-1, 0, 1\}, \quad \forall m \in M, r \in R, q \in Q_m, i \in I_{mq}, \\
& \quad y_r \in \{0, 1\}, \quad \forall r \in R, \\
& \quad AV_t, UV_r \quad \text{integer}, \quad \forall t \in T, r \in R
\end{align*}
\]

- Minimize cost of selected routes
- All demand must load
- Transshipment cargo is loaded on all legs
- Only use available ships
## Master problem – coefficients

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- Constraining the number of ships used
- Ensuring all demand is loaded
- Ensuring all transshipment done correctly
- The cost of the routes
Heuristic – support columns

- One column per transshipment option

- \( L_{mrqi} = 1 \)
  - Direct loading of the cargo

- \( L_{mrqi} = -1 \)
  - Leg of the current transshipment option for the cargo

- Cost
  - The cost per transshipment move in the port of transshipment times the volume of the cargo
Heuristic –
initial columns 1

- All sets with 3 – 20 ports are generated based on following set minimizing initiatives:
  - Min 2 ports per region unless it is a hub
  - Min 1 hub per route
  - Upper limit on number of regions in a route
- The order of the ports is fixed by solving a TSP
- All possible cargo is loaded
  - Direct loading has priority
- Minimal cargo calculated based on the direction of the route
Heuristic – initial columns 2

- The cost is calculated based on
  - Which ship type is used
  - What number of ships is used
  - What speed is used
  - Which ports are included in the route
- Columns added based on
  - Cost
  - Capacity utilization
- If no columns are added, the dummy ship type is used
Heuristic – restricted LP-master

\[
\begin{align*}
\min & \quad \sum_{r \in R} C_r y_r \\
\sum_{r \in R} L_{mrq} y_r & \geq 1, \quad \forall m \in M, q = 0, i = 0, \\
\sum_{r \in R} L_{mrqi} y_r & \geq 0, \quad \forall m \in M, q \in Q_m \setminus \{0\}, i \in I_{mq} \setminus \{0\}, \\
\sum_{r \in R} UV_{tr} y_r & \leq AV_t, \quad \forall t \in T, \\
y_r & \geq 0, \quad \forall r \in R,
\end{align*}
\]

- Set partitioning constraints
- Set covering constraints
- CPLEX used for solving
## Heuristic – column generation

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</table>
Heuristic – the rounding heuristic

- Iterative heuristic
- One fractional variable is rounded up per iteration
- Tie break:
  - Route column > support column
  - Larger number of cargoes
- Stop criteria:
  - All variables are integer
  - Problem becomes infeasible
Future work

- Extend current heuristic:
  - Inclusion of non-simple routes
  - Inclusion of canal transits & restrictions
  - Transit time considerations
- Comparison of small instances
  - Solution quality
  - Time consumption
- Linearize the flow model to find a lower bound for the heuristic.
Routing and scheduling in liner shipping

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